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Bounds on the Higgs-Boson Mass in the Presence of Non-Standard Interactions

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The triviality and vacuum stability bounds on the Higgs-boson mass are revisited in the presence of new interactions parameterized in a model-independent way by an effective lagrangian. When the scale of new physics Λ is below 50 TeV the triviality bound is unchanged but the stability lower bound is increased by $40 \div 60$ GeV. Should the Higgs-boson mass be close to its current lower experimental limit, this leads to the possibility of new physics at the scale of a few TeV, even for modest values of the effective lagrangian parameters.

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a. Introduction In spite of a huge experimental effort, the Higgs particle, the last missing ingredient of the Standard Model (SM) of electroweak interactions has not been discovered yet. For a Higgs-boson mass $m_h \lesssim 115$ GeV the most promising production channel at LEP2 has been $e^+e^- \to Zh$ that provides the limit [1] $m_h > 113.2$ GeV for the SM Higgs-boson. The Higgs particle also contributes radiatively to several well measured quantities and this can be used to derive the upper bound [2] $m_h \lesssim 212$ GeV at 95 % C.L.. Both these limits are, however, highly model-dependent.

There also exist theoretical constraints on m_h based on the so-called triviality and vacuum stability arguments. As it is well know [3] the renormalized ϕ^4 theory cannot contain an interaction term ($\lambda\phi^4$) for any non-zero scalar mass: the theory must be trivial. Within a perturbative approach this statement corresponds to the fact that the running coupling constant $\lambda(\kappa)$ necessarily diverges at a finite value of the renormalization scale κ (the Landau pole). Consequently, only a non-interacting theory is consistent at all energy scales. An analogous effect occurs in the scalar sector of the SM (modified to some extend by the presence of gauge and Yukawa interactions). This, however, does not necessary imply a trivial scalar sector since there is no reason to believe the SM to be valid at arbitrarily high energy scales.

Assuming the SM is an effective theory applicable only below an energy scale Λ, the Landau pole should occur at scale Λ or above, and this condition gives a (Λ-dependent) upper bound on m_h [4]. On the other hand, for sufficiently small m_h radiative corrections can destabilize the ground state. Then the requirement that the SM vacuum is stable for scales below Λ provides a lower bound on m_h [5].

The triviality and vacuum stability bounds on m_h are usually derived assuming SM interactions. However, if the scale of new physics is sufficiently low ($\sim$ a few TeV) one could expect non-standard interactions to generate important modifications of these constraints. The problem deserves special attention given the possibility that the Higgs-boson was observed at LEP2 [6] with a mass $m_h \simeq 115$ GeV; in this case the SM constraint from vacuum stability requires $\Lambda \lesssim \mathcal{O}(100)$ TeV [7] (the precise number depends on the top quark mass), allowing for the attractive possibility that Λ is actually much lower, even at the level of a few TeV.

In this letter we determine the manner in which heavy physics with scales in the 10 TeV region modify the theoretical constraints on the Higgs-boson mass. We follow a model-independent approach parameterizing the heavy physics effects by an effective Lagrangian satisfying the SM gauge symmetries. Then, using standard procedures, we derive the stability and triviality bounds on m_h as a function of the heavy physics scale Λ. Since LHC, the future proton-proton collider, is expected to be sensitive to scales Λ of the order of a few TeV, the results will be presented for scales between 0.5 and 50 TeV.

b. Non-Standard Interactions Our study of the stability and triviality constraints on the Higgs-boson mass will be based on the SM Lagrangian modified by the ad-

dition of a series of gauge-invariant effective operators $\mathcal{O}_i$ whose coefficients α_i parameterize the low-energy effects of the heavy physics [8]. Assuming that these non-standard effects decouple implies [9] that these operators appear multiplied by appropriate inverse powers of Λ. The leading effects are then generated by operators of mass-dimension 6*. Given our emphasis on Higgs-boson physics the effects of all fermions excepting the top-quark can be ignored †. We then have

$$\mathcal{L}_{\rm tree} = -\frac{1}{4}(F^2+B^2) + |D\phi|^2 + i\bar{q}\not{D}q + i\bar{t}\not{D}t \\ + f\left(\bar{q}\tilde{\phi}t + \text{h.c.}\right) - \lambda\left(|\phi|^2 - v^2/2\right)^2 + \sum_i \frac{\alpha_i}{\Lambda^2}\mathcal{O}_i, \quad (1)$$

where ϕ ($\tilde{\phi} = -i\tau_2\phi^*$), q and t are the scalar doublet, third generation left-handed quark doublet and the right-handed top singlet, respectively. D denotes the covariant derivative, $F^i_{\mu\nu}$ and $B_{\mu\nu}$ the $SU(2)$, $U(1)$ field strengths whose couplings we denote by g and g'.

When the heavy interactions are weakly coupled the leading effects at low energy are determined by those α_i generated at tree level by the heavy physics (loop-generated coefficients are suppressed by coupling constants and numerical factors $\sim 1/(4\pi)^2$ [12]). Because of this we will consider only those operators that can be generated at tree-level by the heavy physics. There are 81 dimension-six operators (for one family) [10], but only 16 can be generated at tree-level and involve only the fields in (1). Of these 5 contribute directly to the effective potential, the remaining 11 affect the results only through their RG mixing and, being suppressed by a factor $\sim 1/(G_F\Lambda^2)$, will play a sub-dominant role. We will include only one of these operators to illustrate these effects.

In the calculations below we will include the set

$$\begin{array}{lll} \mathcal{O}_\phi = \frac{1}{3}|\phi|^6 & \mathcal{O}_{\partial\phi} = \frac{1}{2}\left(\partial|\phi|^2\right)^2 & \mathcal{O}^{(1)}_\phi = |\phi|^2\,|D\phi|^2 \\ \mathcal{O}^{(3)}_\phi = \left|\phi^\dagger D\phi\right|^2 & \mathcal{O}_{t\phi} = |\phi|^2(\bar{q}\tilde{\phi}t + \text{h.c.}) & \mathcal{O}^{(1)}_{qt} = \frac{1}{2}\left|\bar{q}t\right|^2 \end{array} \quad (2)$$

where the first 5 operators contribute directly to the effective potential, while $\mathcal{O}^{(1)}_{qt}$ is included to estimate the effects of RG mixing. Note that only $\mathcal{O}_\phi$ contributes at the tree level to the scalar potential:

$$V^{(\rm tree)} = -\eta\Lambda^2|\phi|^2 + \lambda|\phi|^4 - \frac{\alpha_\phi}{3\Lambda^2}|\phi|^6 \quad (3)$$

where $\eta \equiv \lambda v^2/\Lambda^2$.

*Dimension 5 operators violate lepton number [10] and are associated with new physics at very large scales, so we can safely ignore their effects.

†We assume that chirality-violating effective interactions are natural [11], being suppressed by the corresponding Yukawa couplings.

c. Triviality Bound In order to study the high energy behavior of the scalar potential we derive the RG running equations for λ, η and the α_i. This running is also influenced by the gauge and Yukawa interactions, so the full RG evolution requires the β function for all these couplings. Using dimensional regularization in the $\overline{\rm MS}$ scheme, and defining $\bar{\alpha} = \alpha_{\partial\phi} + 2\alpha^{(1)}_\phi + \alpha^{(3)}_\phi$ we find:

$$\begin{aligned}
\frac{d\lambda}{dt} &= 12\lambda^2 - 3f^4 + 6\lambda f^2 - (3\lambda/2)\left(3g^2+g'^2\right) \\
&\quad + (3/16)\left(g'^4 + 2g^2g'^2 + 3g^4\right) \\
&\quad - 2\eta\left[2\alpha_\phi + \lambda\left(3\alpha_{\partial\phi} + 4\bar{\alpha} + \alpha^{(3)}_\phi\right)\right] \\
\frac{d\eta}{dt} &= 3\eta\left[2\lambda + f^2 - \left(3g^2+g'^2\right)/4\right] - 2\eta^2\bar{\alpha} \\
\frac{df}{dt} &= 9f^3/4 - (f/2)\left(8g_s^2 + 9g^2/4 + 17g'^2/12\right) \\
&\quad + 3\eta\alpha_{t\phi} - (f\eta/2)\left(\bar{\alpha} + 3\alpha^{(1)}_{qt}\right) \\
\frac{d\alpha_\phi}{dt} &= 9\alpha_\phi\left(6\lambda + f^2\right) + 12\lambda^2(9\alpha_{\partial\phi} + 6\alpha^{(1)}_\phi + 5\alpha^{(3)}_\phi) \\
&\quad + 36\alpha_{t\phi}f^3 - (9/4)(3g^2+g'^2)\alpha_\phi \\
&\quad - (9/8)\left[2\alpha^{(1)}_\phi g^4 + \left(\alpha^{(1)}_\phi + \alpha^{(3)}_\phi\right)(g^2+g'^2)^2\right] \\
\frac{d\alpha_{\partial\phi}}{dt} &= 2\lambda\left(6\alpha_{\partial\phi} - 3\alpha^{(1)}_\phi + \bar{\alpha}\right) + 6f\left(f\alpha_{\partial\phi} - \alpha_{t\phi}\right) \\
\frac{d\alpha^{(1)}_\phi}{dt} &= 2\lambda\left(\bar{\alpha} + 3\alpha^{(1)}_\phi\right) + 6f\left(f\alpha^{(1)}_\phi - \alpha_{t\phi}\right) \\
\frac{d\alpha^{(3)}_\phi}{dt} &= 6(\lambda + f^2)\alpha^{(3)}_\phi \\
\frac{d\alpha_{t\phi}}{dt} &= -3f(f^2+\lambda)\alpha^{(1)}_{qt} + (15f^2/4 - 12\lambda)\alpha_{t\phi} \\
&\quad - (f^3/2)\left(\alpha_{\partial\phi} - \alpha^{(1)}_\phi + \bar{\alpha}\right) \\
\frac{d\alpha^{(1)}_{qt}}{dt} &= (3/2)\alpha^{(1)}_{qt}f^2
\end{aligned} \quad (4)$$

where we neglected terms quadratic in the α_i, and $t \equiv \log(\kappa/m_Z)/(8\pi^2)$, κ being the renormalization scale. The evolution of the gauge couplings g, g' and g_s (for the strong interactions) are unaffected by the α_i.

In order to solve the equations (4) we have to specify appropriate boundary conditions. For the SM parameters these are determined by requiring that the correct physical parameters are obtained at the electroweak scale. The values of λ, f and η at $t=0$ are fixed using the physical Higgs-boson mass [7], the top mass and the scalar field vacuum expectation value. Since the experimental errors in the top-quark mass are larger than the expected deviations from the tree-level expression we use $m_t = v_0 f/\sqrt{2} = f \times 174$ GeV for simplicity. We also require that the solutions to (4) reproduce the correct electroweak vacuum where the scalar field has the expectation value $\langle\bar{\varphi}\rangle \simeq v_0/\sqrt{2}$ ‡. The relation between

‡ We ignored a small correction to the W mass $\propto \alpha^{(1)}_\phi (v_0/\Lambda)^2$

the SM tree-level vacuum v and the physical electroweak vacuum v_0 is

$$v_0 = v - \frac{1}{4\lambda(0)v_0^2}\left.\frac{\partial(V_{\rm eff} - V_{\rm SM}^{(\rm tree)})}{\partial(\bar\varphi/\sqrt{2})}\right|_{\bar\varphi = v_0/\sqrt{2}}, \qquad (5)$$

where $V_{\rm SM}^{(\rm tree)}$ denotes the tree-level SM potential, $V_{\rm eff}$ the effective potential calculated up to 1-loop including all effective operator contributions, and $\lambda(0)$ the running coupling constant evaluated at $t = 0$. Finally we require that the gauge coupling constants satisfy $g(0) = 0.648,\ g'(0) = 0.356,\ g_s(0) = 1.218$.

The boundary conditions for the α_i are naturally specified at the scale $\kappa = \Lambda$. For the weakly-coupled heavy interactions considered here it is natural to assume that $\alpha_i|_{\kappa=\Lambda} \lesssim \mathcal{O}(1)$ [12] (the triviality bounds are insensitive to the precise value). In Fig. 2 (b),(c) we have plotted two examples of the solutions to (4) using the above boundary conditions.

The triviality bound on m_h will be obtained by requiring λ and α_ϕ to remain below specified values (as opposed from requiring an actual divergence)

$$\lambda(t) \le \lambda_{\rm max} \quad |\alpha_i(t)| \le \alpha_{\rm max} \qquad (6)$$

for all scales below Λ. We will present results for $\alpha_{\rm max} = 1.5$ and $\lambda_{\rm max} = \pi$ and $\pi/2$ (this result is insensitive to the choice of $\alpha_{\rm max}$). The bound obtained by saturating either of the above inequalities is plotted in Fig. 1 (a). We note that the results are almost identical to the ones obtained for the SM [4].

In order to understand qualitatively the absence of significant corrections to the triviality bound, it is useful to switch off all α_i except α_ϕ. Then, since $d\ln(\alpha_\phi)/dt > 0$[§], $|\alpha_\phi|$ will decrease when evolving from the scale Λ downwards, reaching values $\sim 10^{-1} \div 10^{-2}$ at $\kappa = m_Z$ (for $|\alpha_\phi(\Lambda)| \sim 1$). In addition, note also that the effects of α_ϕ on the evolution of the SM parameters are suppressed by a factor of η. Hence the effects of α_ϕ on the RG evolution of λ are screened, leaving the SM triviality bound essentially unaffected.

d. Vacuum Stability Bound In order to investigate the vacuum structure of the effective theory we will first calculate the effective potential:

$$V_{\rm eff}(\bar\varphi) = -\sum_n \frac{1}{n!}\Gamma^{(n)}(0)\bar\varphi^n, \qquad (7)$$

where $\Gamma^{(n)}(0)$ are n-point 1PI vertices at zero external momenta and $\bar\varphi$ is the classical scalar field. Using the Landau gauge[**] we find:

$$V_{\rm eff}(\bar\varphi) = V^{(\rm tree)} + \frac{1}{64\pi^2}\sum_{i=0}^{5} c_i R_i^2[\ln(R_i/\kappa^2) - \nu_i] \qquad (8)$$

where $c_0 = -4,\ c_1 = 1,\ c_{2,4} = 3,\ c_3 = 6,\ c_5 = -12$, $\nu_{0,1,2,5} = 3/2,\ \nu_{3,4} = 5/6$, $R_0 = \eta\Lambda^2$ and

$$\begin{aligned}
R_1 &= \lambda(6|\bar\varphi|^2 - v^2)\left[1 - (2\alpha_{\partial\phi} + \alpha_\phi^{(1)} + \alpha_\phi^{(3)})|\bar\varphi|^2/\Lambda^2\right] \\
&\quad -5\alpha_\phi|\bar\varphi|^4/\Lambda^2 \\
R_2 &= \lambda(2|\bar\varphi|^2 - v^2)\left[1 - (\alpha_\phi^{(1)} + \alpha_\phi^{(3)}/3)|\bar\varphi|^2/\Lambda^2\right] \\
&\quad -\alpha_\phi|\bar\varphi|^4/\Lambda^2 \\
R_3 &= (g^2/2)|\bar\varphi|^2\left(1 + |\bar\varphi|^2\alpha_\phi^{(1)}/\Lambda^2\right) \\
R_4 &= [(g^2 + g'^2)/2]|\bar\varphi|^2\left(1 + |\bar\varphi|^2(\alpha_\phi^{(1)} + \alpha_\phi^{(3)})/\Lambda^2\right) \\
R_5 &= f|\bar\varphi|^2\left(f + 2\alpha_{t\phi}|\bar\varphi|^2/\Lambda^2\right),
\end{aligned}$$

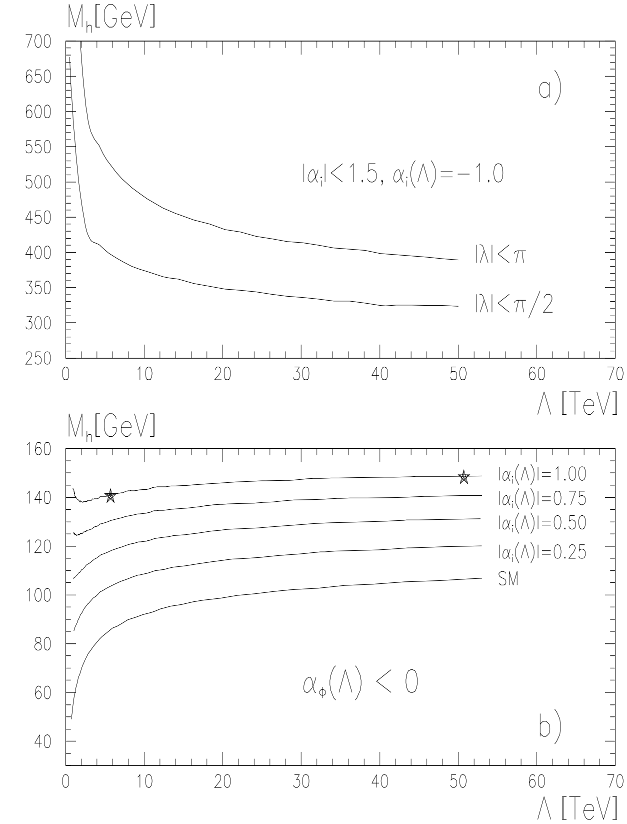

FIG. 1. *(a) Triviality bound on the Higgs-boson mass obtained from (6). (b) The stability bound on m_h obtained from (9) when $\alpha_\phi(\Lambda) < 0$, $m_t = 175$ GeV. The stars correspond to the solutions (1) and (2) of Fig. 2.*

which modifies the relationship between G_F and v_0 at the 10% level; changing the bounds on m_h by $\lesssim 6\%$.

[§] Here we consider heavy Higgs bosons, therefore λ remains positive in the whole integration region.

[**] The loop contributions to $V_{\rm eff}$ are gauge dependent [13], yet since the RG-improved tree-level effective potential is gauge-invariant, the stability bound depends weakly on the gauge parameter leading to an uncertainty $\Delta m_h \lesssim .5$ GeV [7].

It is understood that the above expression is accurate up to corrections of order $1/\Lambda^4$.

The *form* of the effective potential is precisely the same as the one in the pure SM, the whole effect of the effective operators can be absorbed in a re-definition of the R_i. We also include the scale dependence of $\bar{\varphi}$: $\bar{\varphi}(t) = \exp\left\{-\int_0^t \gamma dt'\right\} \bar{\varphi}(t=0)$, where $\gamma = 3f^2/2 - 3(3g^2 + g'^2)/8 - \eta\bar{\alpha}/2$ (the couplings appearing in γ are understood to be the solutions to (4)). In the following we will consider the RG improved effective potential $V_{\rm eff}(\bar{\varphi}(t))$ defined using (8) and $\bar{\varphi}(t)$.

We note that $V_{\rm eff}(\bar{\varphi}(t))$ is scale invariant, that is, $\kappa\ dV_{\rm eff}(\bar{\varphi}(t))/d\kappa = 0$ (to one loop and ignoring terms quadratic in the α_i). In verifying this relation the constant term in (8) must be chosen appropriately, our choice is determined by the requirement $V_{\rm eff}(\bar{\varphi} = 0) = 0$, which is consistent with (7); for details see Ref. [15].

When using the above expressions to derive the stability bound on m_h we will need to consider values of $\bar{\varphi}$ substantially larger then the electroweak scale v_0. Therefore we shall choose a renormalization scale $\kappa \sim \bar{\varphi}$ in order to moderate the logarithms that appear in $V_{\rm eff}$.

Fig. 2 illustrates the behaviour of the effective potential renormalized at the scale $\kappa = \bar{\varphi}$. To show the relevance of RG running of effective-potential parameters we also plot the evolution of λ and α_ϕ for two sets of initial conditions corresponding to the (m_h, Λ) values marked in Fig. 1 by stars. As it is seen from the figure effects of the running are substantial, e.g. for the set (2) λ changes by almost 100% while α_ϕ by more than 200% and undergoes a sign change. This emphasises the fact that the RG running of the coefficients α_i must be included when studying the vacuum stability of the system††.

The initial conditions for the running couplings guarantee that the electroweak vacuum is at $\langle\bar{\varphi}\rangle = v_0/\sqrt{2}$. However if $V_{\rm eff}$ at some large value of the field $\bar{\varphi}_{\rm high}$ is smaller than $V_{\rm eff}(\langle\bar{\varphi}\rangle)$ this vacuum becomes unstable (as there would be a possibility of tunnelling‡‡ towards the region of lower energy). This will occur when the Higgs-boson mass is sufficiently small (corresponding to a small value of $\lambda(0)$), and provides a lower bound on m_h. In this case $\bar{\varphi}_{\rm high}$ defines a scale at which the theory breaks down, so that $\bar{\varphi}_{\rm high} \sim \Lambda$. In actual calculations we took $\bar{\varphi}_{\rm high} = 0.75\Lambda$ since (1) is valid for scales below Λ, hence the stability bound on m_h is determined by the condition

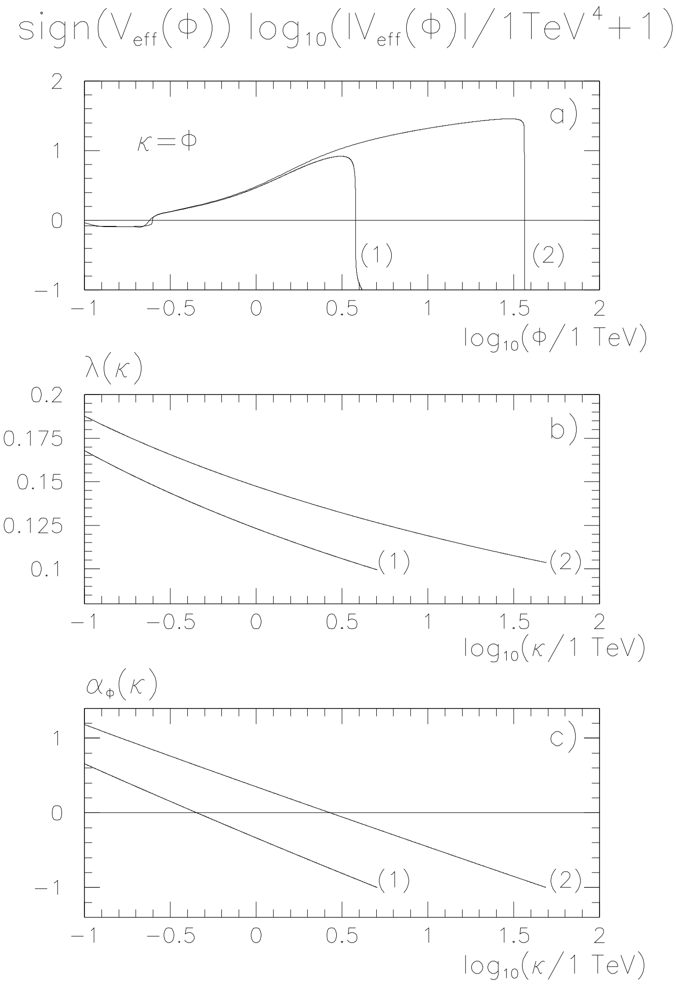

FIG. 2. *The effective potential renormalized at the scale* $\kappa = \phi$ *(a); (we have plotted* $\mathrm{sgn}(V_{\rm eff})\log_{10}[(V_{\rm eff}/1\ TeV^4) + 1]$ *in order to make visible the shallow minimum at* $\bar{\varphi} = v_0/\sqrt{2}$*). The running of* λ *(b) and* α_ϕ *(c) when* $\alpha_i(\Lambda) = -1$, $m_t = 175$ *GeV, for* $\Lambda = 5.1$ *TeV,* $m_h = 140.4$ *GeV (curves (1)) and* $\Lambda = 48.9$ *TeV,* $m_h = 148.7$ *GeV (curves (2)).*

$$V_{\rm eff}(\bar{\varphi} = 0.75\Lambda)|_{\kappa=0.75\Lambda} = V_{\rm eff}(\bar{\varphi} = v_0/\sqrt{2})|_{\kappa=v_0/\sqrt{2}} \quad (9)$$

where, as mentioned previously, we have chosen the renormalization scale κ to tame the effects of the logarithmic contributions to $V_{\rm eff}(\bar{\varphi})$. The resulting bound on m_h as a function of Λ for various choices of $\alpha_i(\Lambda)$ is plotted in Fig. 1 (b).

In obtaining the stability bounds of Fig. 1 (b) we assumed all couplings α_i had the same magnitude at the high scale Λ, and $\alpha_\phi < 0$ (the results are insensitive to the sign of the other α_i). For other values of α_i we found that when $\Lambda > 300$ GeV there is a curve in the $\alpha_\phi - \alpha_{t\phi}$ plane below which either $\bar{\varphi} = 174$ GeV is not a minimum or, if it is, then there is another deeper minimum at a scale 174 GeV $< \bar{\varphi} < \Lambda$; we can roughly say that this

††Some corrections to the SM vacuum stability bound due to $\mathcal{O}_\phi$ were discussed in [17]. However, the authors did not consider one-loop contributions to the effective potential generated by $\mathcal{O}_\phi$, nor the RG running of α_ϕ.

‡‡The question of the tunnelling rate will not be discussed in this latter, we shall assume that tunnelling is possible with the transition time smaller than the age of the universe.

unphysical scenario can be avoided if $\alpha_\phi \lesssim -0.1$ [§§].

There is an important remark here in order. The SM vacuum stability bound together with the experimental limit $m_h > 113.2$ GeV implies $\Lambda \lesssim \mathcal{O}(100)$ TeV. Assuming now that the limit on m_h remains unchanged in presence of effective operators [16,17], then, as seen from Fig. 1 (b), even for the modest values $|\alpha_i| = 0.25,\ 0.50,\ 0.60$ the upper bound on Λ is significantly reduced to $\Lambda \simeq 20,\ 4\ 1$ TeV, respectively!

e. Summary and Conclusions We have considered the triviality and stability restrictions on m_h when the SM is the low-energy limit of a weakly-coupled decoupling theory of typical scale Λ. It was assumed that there is a significant gap between Λ and the typical experimental energies so that the heavy interactions can be accurately described by a set of effective vertices. We showed that for the scale of new physics in the region 0.5 TeV $\leq \Lambda \leq 50$ TeV the SM triviality (upper) bound remains unmodified. In contrast stability (lower) bound could be increased by $40-60$ GeV reducing substantially the allowed region of m_h values. If m_h is close to its current experimental limit, then the maximum allowed value of Λ would be decreased dramatically even for modest values of coefficients of effective operators, implying new physics already at the scale of a few TeV.

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[§§] We do not expect this result to be modified significantly when terms of order $1/\Lambda^4$ are included: a contribution $\sim \alpha^{(8)}\bar{\varphi}^8/\Lambda^4$ can balance the destabilizing effect of $\mathcal{O}_\phi$ only when $\bar{\varphi} \sim \Lambda$ which again leads to $\Lambda \sim 300$ GeV.